COMMENT ON THE FIRST VERSION

The first version of the paper posted in April 2021 and found on https://arxiv.org/abs/2104.05109v1 contained a serious mistake. The way my retraction is phrased in https://arxiv.org/abs/2104.05109v2 is itself a bit misleading, as it does not point to the core of the problem. Let me explain more precisely what was flawed. The only serious mistake is on page 30, line 5, when asserting that:

"The flow $\tau_{t,\theta}^{(T)}$ clearly corresponds to a localized translation in the direction $\vec{\theta}$ and the conclusions of Lemma 5.3 hold with the obvious modifications."

Let us recall that the desired properties of a localized translation are stated in Lemma 5.3 at the bottom of page 29, we want:

(1) A volume-preserving diffeomorphism,
(2) that coincides with a translation near the origin...
(3) ..and with the identity far away.

The construction of [Geo99] does provide a localized translation $\tau_t^{(T)}$ in a given direction $\vec{u}$, this is fine and Lemma 5.3. is valid. Writing τ_t as $\tau_t = \mathrm{Id} + \psi_t$ is of course possible, and considering $\tau_{t,\theta} = \mathrm{Id} + \rho_\theta \circ \psi_t$ where ρ_θ is the rotation by θ does define some diffeomorphism of the plane that coincides with a translation by $t\vec{\theta}$ near the origin and with the identity far away. **But it is <u>not</u> measure-preserving for $\theta \neq 0$.** This is a fatal issue, as we use the fact that all the $\tau_{t,\theta}$'s are measure-preserving repeatedly below in the proof of Proposition 5.2 (a manifestation of that problem can be seen around (5.10) when we use a convexity inequality that is ultimately converted into an equality. This is not impossible, but it is dubious. We are grateful to Eric Thoma for pointing this to us early on.) and because Proposition 5.2 is absolutely crucial in the strategy.

On the one hand, if one considers instead the correct, measure-preserving version of the "rotated" flow, then the identities used in Claim 5.4 fail to hold. On the other hand, one can try to take the Jacobian of $\mathrm{Id} + \rho_\theta \circ \psi_t$ into account. Both attempts are ultimately create error terms that are bounded, but not small.

The workaround, as presented in the current version (https://arxiv.org/abs/2104.05109v3) consists in changing the construction of the localized translation. Instead of relying on Georgii's result, we build a localized translation of the form $\mathrm{Id} + \psi$ that takes a very long time $e^{1/\epsilon}$ to interpolate between being a translation near the origin and being the identity far away. It allows us to have $\int |\psi|^2$ of order ϵ (whereas the same quantity is of order 1 if one uses the construction of Georgii). On the other hand, we can show that the effect on the Coulomb energy of a localized translation $\mathrm{Id} + \psi$ is (roughly speaking) of order $\int |\psi|^2$. We may thus build localized translations that have an arbitrarily small energy cost, which puts us back in business.

References

[Geo99] Hans-Otto Georgii. Translation invariance and continuous symmetries in two-dimensional continuum systems. In *Mathematical results in statistical mechanics (Marseilles, 1998)*, pages 53–69. World Sci. Publ., River Edge, NJ, 1999.